Lecture - 6

Outline: zero sound as density-density response
variational wfⁿ for fermi liquid,
RG approach to fermi liquid.

Zero sound as a density-density response

$\langle \rho(q,\omega)\,\rho(-q,-\omega)\rangle$

$\int \underbrace{\mu(q,\omega)}_{\text{density term in } \mathcal{H}}\, \rho(q,\omega)$

so

$\delta\rho \sim \chi\mu$

$\chi \to$ susceptibility

$\chi \sim \int [\rho_{r t}, \rho_{(0,0)}]\, e^{iqr - i\omega t}$ real time

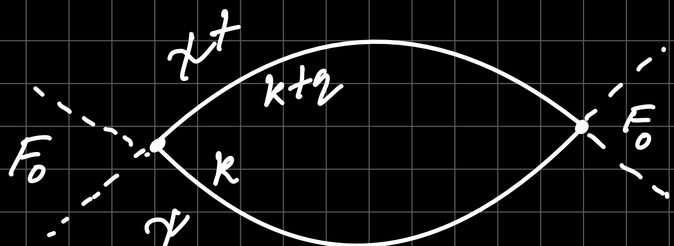
$\downarrow$

$\langle \rho(it,\omega), \rho(-it,-\omega)\rangle$

$$\rho(r,t) = \psi^{+}(r,t)\,\psi(r,t)$$

$$\rho(q,\omega) \sim \int_{k,\nu} \psi^{+}(k,\nu)\,\psi(k+q,\omega+\nu)$$

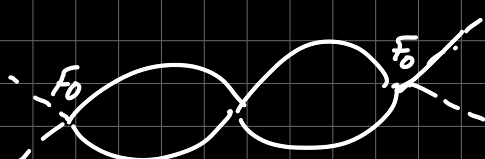
density density resp. in diagrammatically



$\hookrightarrow I$

(Lindhard susceptibility for fermi liquids)

$+$



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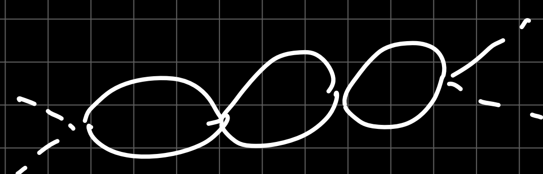


$$\chi = \langle \rho \rho \rangle = I + I^2 F_0 + I^3 F_0^2 - - - -$$

$$= I \left[\frac{1}{1 - I F_0} \right] \quad — (1)$$

$$I \sim \left\{ \frac{\omega}{2 v_F} \log \left[\frac{\omega + v_F q}{\omega - v_F q} \right] - 1 \right\}$$ (Lindhard f^n, non-interacting susceptibility)

$\hookrightarrow$ in 211B

relation to ising model (i.e. susceptibility)

$$\mathcal{H} = -J \sum s_i s_j - h \sum s_i \qquad \chi(J=0) \sim \frac{1}{T}$$

[what are all the thermo resp. def^{n}s]

$$\mathcal{H}_{MF} = \sum -(Jzm + h) s_i$$

$\langle s_i \rangle = m$

$$= \chi_0 h_{eff} = \chi_0 [Jzm + h] = \frac{\chi_0 h}{1 - \chi_0 J z}$$

$\hookrightarrow$ str. of int.

$\Downarrow$

compare with (1)

MFT $\equiv$ summing ∞ # of diagrams

Poles of χ determine int. excitations i.e. modes that can dominate response even under slight perturbation.

$$\frac{1}{F_0} = \frac{\omega}{2 v_F q} \log \left(\frac{\omega + v_F q}{\omega - v_F q} \right) - 1$$

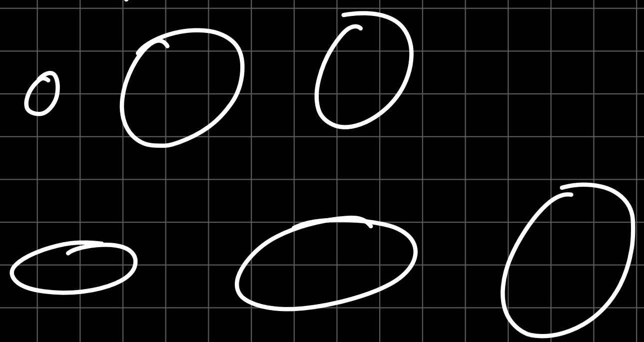
$$\lambda = \frac{\omega}{v_F q}$$

$$\frac{1}{F_0} = \frac{\lambda}{2} \log \left(\frac{1 + \lambda}{\lambda - 1} \right) - 1$$

[in Ising model, poles of sus. = give T_c, locat^n where all response functions are diverging]

Check:- Pines & Nozieres

\# Can have possibly ∞ # of sound modes.



$\Rightarrow$ why doesn't happen in He^3?

for $l=0$ $F_0>0$ for undamped

[for $l=2$ $F_1>6$ for undamped]

not satisfied in He^3

Variational Wfn. for fermi liquid

Hubbard model:

$$H = \underbrace{-t\sum_{\langle ij\rangle}[c_{i\sigma}^{\dagger}c_{j\sigma} + h.c.]}_{H_t} + \underbrace{U\sum_i n_{i\uparrow}n_{i\downarrow}}_{H_U}$$

$$|\psi_0\rangle \text{ or } |\psi_{exact}\rangle = \lim_{\beta\to\infty} e^{-\beta H}|\text{product state}\rangle$$

$$|\psi_{var}\rangle = e^{-\frac{\beta}{2}H_U}\, e^{-\frac{\beta}{2}H_t}\,|\text{product state}\rangle$$

$\beta_2\to\infty$ leads to

↳ is a good approx^n to actual wavef^n (why?)

simplificat^n of trotter decomposition.

$$= g^{U\sum n_{i\uparrow} n_{i\downarrow}} |\Psi_0\rangle \qquad \left(= \prod_{\substack{k\sigma \\ |k|<k_F}} c^\dagger_{k\sigma} |0\rangle \right)$$

a free parameter

$$E(g) = \langle \Psi(g) | H | \Psi(g) \rangle$$

(what is g ?)

min $E(g)$ wrt g

"Gutzwiller projected wf^n"

$0 < g < 1$

$g = 1$: Metal

$g = 0$: insulator

prohibit double occupancy

$$g^{\sum n_{i\uparrow} n_{i\downarrow}} | \underbrace{\uparrow\downarrow \ \ \uparrow \ \ \downarrow}_{|\Psi_0\rangle} \ \ \uparrow \rangle = g |\Psi_0\rangle$$

$$|\Psi_0\rangle = |\underline{\uparrow\downarrow} \ \ \underline{\uparrow} \ \ \underline{\uparrow} \ \ \downarrow\rangle + |\underline{\downarrow} \ \ \underline{\uparrow\downarrow} \ \ \underline{\uparrow} \ \ \underline{\downarrow\uparrow}\rangle + |\downarrow \ \ \uparrow \ \ \downarrow \ \ \uparrow\rangle + \cdots$$

$$\lim_{g\to 0} g^{\sum_i n_{i\uparrow} n_{i\downarrow}} |\Psi_0\rangle = |\downarrow \uparrow \downarrow \uparrow\rangle$$

purely spin wf^n

Fermi liquid — Mott insulator

$0 < g < 1$ — $g = 0$

In 1d, this can describe Luttinger liquid.

Gutzwiller approximation is diff from Gutzwiller projection.

$$\frac{\langle\psi|E|\psi\rangle}{V}(q,d) = \frac{\langle\psi|E|\psi\rangle}{V}(q^*,d)$$

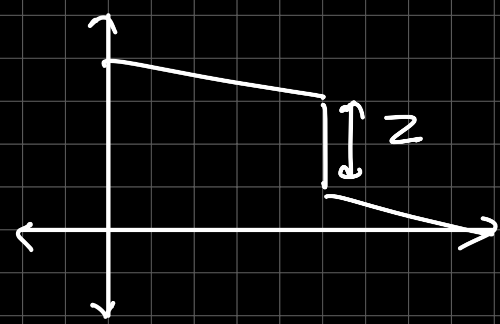
d : # of doubly occupied states

$$= Ud + \sum_{\sigma=\uparrow\downarrow} q_\sigma \bar{\varepsilon}_\sigma$$

$$\bar{\varepsilon}_\sigma = \frac{\sum_{k<k_F} \varepsilon_k}{V}$$

$q = Z$

$q \sim 8d(1-2d)$



$\Downarrow$ min. wrt d

$$d = \frac{1}{4}\left[1 - \frac{U}{U_c(t)}\right]$$

for $U \to U_c$, $d \to 0$

$$q = 1 - \left(\frac{U}{U_c}\right)^2$$

$\to 0$ or $Z \to 0$

caricature of FLT $\to$ Mott insulator phase

Modern perspective

$c_\sigma = b\, f_\sigma$

b: only charge; f_σ: spin only

(parton / slave-fermion approach)

Fermi liquid : b is in a superfluid phase

f is in a free fermion state

$$\langle c_k^+ c_k\rangle \sim \underset{\substack{\parallel \\ Z}}{|b|^2} \langle f_k^+ f_k\rangle$$

$|b|^2$ $\to$ order parameter for superfluid phase

$\therefore$ Z, an OP, is related to a venerable OP i.e. superfluid OP.

$b \to b e^{i\theta}$ $f \to f e^{-i\theta}$ $\Rightarrow$ asking for Gauge fields to appear

$$|(\partial - a)b|^2 + |(\partial + a)f|^2 + (\nabla\times a)^2 + \ldots$$

u(1) gauge field

gauge field that is allowed.

$$\ldots + (b^\dagger b)(f^\dagger f) + \ldots$$

$b, f \to$ gauge charges $+1, -1$

$\Rightarrow$ so in a FL, b condenses & due to higgs effect, a gets gapped out. $\Rightarrow$ so no such low en. photon excitations.

Gutzwiller approxm. can be thought of as resulting from parton approach.

Improving gutzwiller approximatn using this approach.

$$|\psi\rangle \sim e^{-f(r_i - r_j)} |\psi_0\rangle$$

[more & more comments] $\Rightarrow$

b: mott insulator

f: fermi surface

Fractionalized phase with charge gap & Fermi surface

* Fermi liq. as a higgs phase.

Renormalization

Ex 1: Ising model

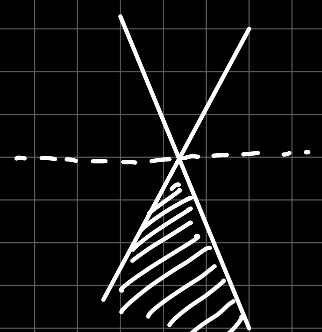
$$\int d^dx \; (\nabla\phi)^2 + U\int d^dx \; \phi^4(x)$$

$$\langle \phi(x)\phi(0)\rangle \sim \frac{1}{x^{d-2}} \qquad \phi(L)\phi(0) \sim \frac{1}{L^{d-2}}$$

$$U L^d \cdot \frac{1}{L^{\left(\frac{d-2}{2}\right)4}} = U L^{(4-d)}$$

$d>4$
vanishes

Ex 2: Graphene



$$\int d^dx \, d\tau \, [\bar\psi \not\partial \psi]$$
$$+ \int d^dx \, d\tau \; \underbrace{(\bar\psi\psi)^2}_{}$$

$\Downarrow$

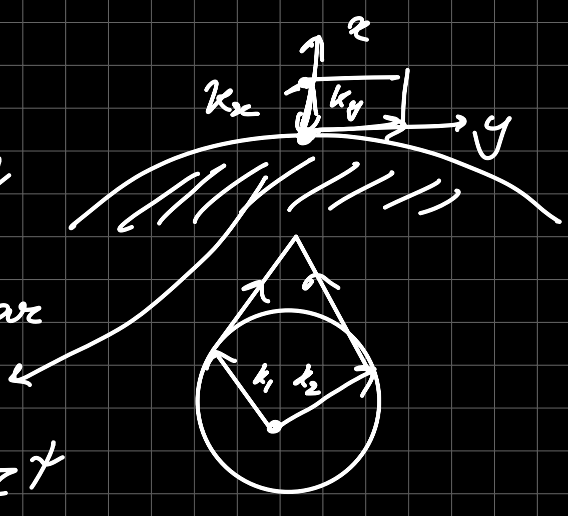
$$\psi\psi \sim \frac{1}{L^d}$$

int. strength at long dist. $\left\{ \dfrac{1}{L^{2d}} \cdot L^{d+1} = L^{1-d} \right.$

if $d>1$: int. are irrel.

if $d=1$: marginal
$\Downarrow$
Luttinger Liquid.

\# $\exists$ multiple ways to do fermion RG



$$\int d\tau \, dk_x \, dk_y \; \psi^\dagger \left(\frac{k^2}{2m} - \frac{k_F^2}{2m}\right)\psi$$

takes into account only small scattering i.e. scattering only near this pnt

$$= \int \psi^\dagger \left(k_x v_F + \frac{k_y^2}{2m}\right)\psi + \int \psi^\dagger \partial_\tau \psi$$

$$= \int d\tau\, dx\, d^{d-1}y \;\; \psi^* \left[\partial_\tau - i v_F \partial_x - \frac{\partial_y^2}{2m} \right] \psi = S$$

<u>scaling</u> <u>arg</u>:-

$$\underbrace{x' = \frac{x}{b^2} \quad \tau' = \frac{\tau}{b^2} \quad y' = \frac{y}{b} \quad \psi' = \psi\, b^{\left(\frac{d+1}{2}\right)}}_{d}$$

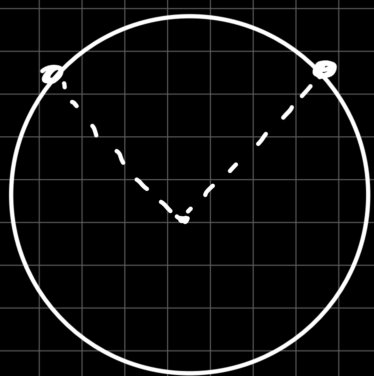
$b > 1$ under this S remains invariant

$$u \int d\tau\, dx\, d^{d-1}y \;\; \psi^\dagger \psi^\dagger \psi \psi \rightarrow u\, b^2 b^2\, b^{d-1}\, b^{-2(d+1)} \times$$

(in ' coordinates)

$$\rightarrow u\, b^{1-d} = u'$$

for $d > 1$ $u' < u$ (for low L scattering)
↳ at one patch



for 2 diff. patch $\Rightarrow u' = u$ ($\neq$ BA)

(Landau didnt consider 2 diff. patches)

*no BEC in 1d

*mermin wagner type of argument for FLT, <u>KT phase</u>

<u>Parton</u> <u>construction</u>:- T. Senthil ("Theory of continuous Mott transitions")